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Performance of the *k*-ε Model in Computation of Asymmetric Turbulent Near Wakes

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Introduction

PERFORMANCE of the *k*-ε turbulence model as used to compute two-dimensional turbulent wakes has been examined by calculating symmetric and asymmetric wakes in nominally zero-pressure gradients in the absence of such complicating parameters as pressure gradient and curvature. Wakes with small asymmetry are computed accurately, but the accuracy diminishes for strong asymmetry even in the near wake region. It is found that it is due to increasing importance of the turbulent transports in strongly asymmetric flows, which the *k*-ε model does not represent well, particularly the counter-gradient diffusion found in strongly asymmetric wake flows.

Calculation Method and Test Cases

The standard *k*-ε method of Launder and Spalding[1] with the standard values of constants has been used to assess its performance. In the present test cases, the pressure variation is small and can be considered to be given; hence, a parabolic method is used starting the calculation at the trailing edge of the wake-producing model. The initial conditions are taken from the experimentally

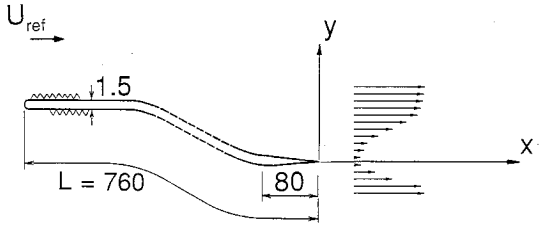

Fig. 1 Configuration of test flows.

Table 1 Characteristics of the test flows at $x = 0$

	Symmetric, mm	Case A, mm	Case B, mm	Case C, mm
δ_{995}, upper side	20	60	34	88
δ_{995}, lower side	20	12	25	15
θ_t	4.7	6.8	5.0	11.3
C_f, upper side	0.0035	0.0033	0.0043	0.0027
C_f, lower side	0.0036	0.0034	0.0030	0.0038
Re_θ, upper side	1840	7420	4200	13800
Re_θ, lower side	1840	1647	3300	1210

Received Jan. 26, 1993; revision received Feb. 28, 1994; accepted for publication March 12, 1994.

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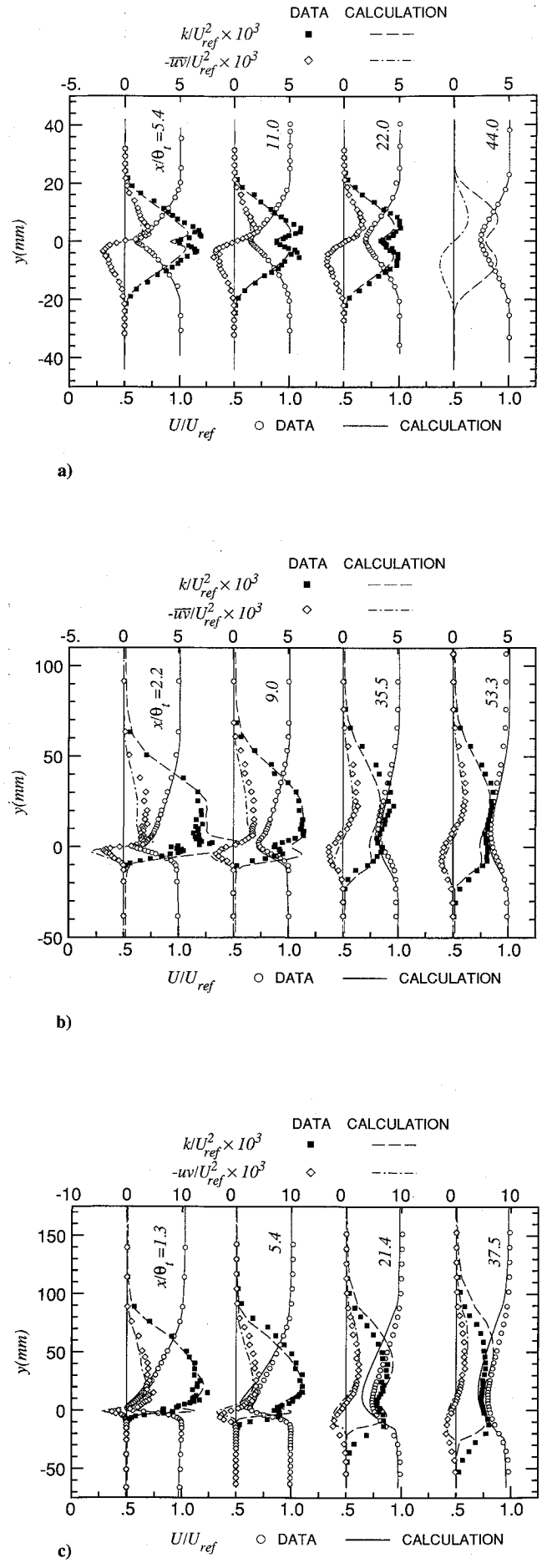

Fig. 2 Comparison of calculation and experimental data: a) symmetric wake, b) asymmetric wake case A, and c) asymmetric wake case C.

measured profiles of velocity components U and V, the turbulent kinetic energy k, and the dissipation rate of the turbulent energy ε, which is deduced from the equilibrium relation $\varepsilon = (0.3k)^{1.5}/l$. The values of the mixing length l are calculated from the data using its definition in terms of the mean-velocity gradient and the shear stress. The boundary conditions are that U at the wake edges is equal to the measured edge velocity U_e measured in the experiment, k and ε satisfy the zero-gradient conditions, $U_e\, dk/dx = -\varepsilon$ and $U_e\, d\varepsilon/dx = C_{\varepsilon 2}\varepsilon^2/k$, where $C_{\varepsilon 2}$ is one of the model constants. The configuration of the flow for which the calculation is made is depicted in Fig. 1. The wake-generating model is a flexible plate whose shape is varied to produce different pressure gradients on the upper and lower sides of the plate controlling the properties of the initial wake. The characteristics of the test flow at the trailing edge including the boundary layer thickness, the friction coefficient C_f, and the momentum thickness Reynolds number Re_θ are shown in Table 1. The details of the experiments and the results are given in Nakayama and Kreplin.[2] The step size in the calculation is initially taken about 0.5×10^{-3} times the momentum thickness θ_t at the trailing edge and is doubled at every 50th step until the step size of $0.2\theta_t$ is reached. About 1000 integration steps are needed to cover a distance of about $40\theta_t$ from the trailing edge, which is considered a near wake region.[3]

Results

The results of the calculation have been compared with the experimental data. Comparison of the developments of the mean velocity U, the turbulent kinetic energy k, and the turbulent shear stress $-\overline{uv}$ for the symmetric case, asymmetric case A, and asymmetric case C are shown in Figs. 2a, 2b, and 2c, respectively. The streamwise distance x is normalized by θ_t. In the symmetric wake case, the calculation results agree well with the experiment except for a minor underprediction of k and shear stress peaks. In the asymmetric case A, the mean velocity and k are predicted fairly well, although the shear stress on the thicker side are consistently underpredicted. In the strongly asymmetric case C, however, it is seen that the mean velocity profile is grossly underpredicted. The energy k is overpredicted on the thicker side and grossly underpredicted on the thinner side. It is seen that this is due to the slow growth rate calculated on the thinner side and faster spreading rate on the thicker side. Patel and Scheuerer[4] made similar a calculation for symmetric wakes and asymmetric wake generated by roughness on one side of a plate model and found relatively good agreement in the near wake region but prediction deteriorated downstream. They indicated that one reason for the poor performance of the k-ε model in wakes is due to its lack of representing the effects of the intermittency. Cho and Chung[5] incorporated the intermittency effects by their k-ε-γ model but a major improvement was not found. The present calculation indicates that even in near wakes, the prediction is poor when the asymmetry is severe. The data indicate that the size of turbulence on the thicker side is so large that the large eddies from this side also influence the spreading rate of the thinner side. The k-ε model which estimates the scale from the local values only cannot reflect the nonlocal effects of the large eddies. In test case C, these nonlocal effects are so strong that there is a transport of momentum and Reynolds stress in the direction opposite of the gradient.[2] The k-ε model fails to capture this phenomenon and is a reason for the poor performance in such highly asymmetric flows. The full paper contains evidence for this as well.

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Time-Accurate Local Time Stepping Method Based on Flux Updating

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I. Introduction

TRADITIONALLY, a globally determined time step has been used in the prediction of unsteady compressible flows via explicit time-marching methods. The time increment established in this way, which is the smallest anywhere in the whole domain that satisfies the stability criteria, implies that for most of the cells only a fraction of their locally allowable time step is used to integrate the solution in time. Obviously, this is a waste of computational effort that should be avoided, especially when the ratio of local time steps spans many orders of magnitude, such as it does in problems requiring a highly variable node spacing and/or involving various scales of temperature. Moreover, the problem is in a deeper layer still: accuracy. For example, for a linear scalar equation solved with a first-order upwind scheme, a Courant-Friedrichs-Lewy (CFL) number equal to one leads to an exact representation of the original equation as higher-order error terms disappear, whereas in fact smaller CFL numbers result in a more diffusive solution.[1]

These simple facts tell us that to improve time-marching schemes for unsteady flow computations with regard to both computational effort and solution accuracy, a variable grid and physics-dependent time step size should be used that is as close as possible to the maximum time step dictated by the local CFL condition. Osher and Sanders[2] were among the first to propose a local time step (LTS) technique using a multistep predictor-corrector method. Their method is based on a grid refinement in time. For a grid that is refined by an integer of r in time, the scheme requires the storage of r-intermediate values, since an r-step predictor-corrector method is used to determine interface values. To overcome this drawback, Berger[3] has introduced interface equations to determine the interface values and to maintain the global conservation. With these methods, the assigned local time step distribution is fixed until the largest time step has been reached without consideration of any possible changes within this time period. This method of handling the problem may not be able to capture some rapidly incoming information at the proper time. In the work of Pervaiz and Baron[4] on reacting flow computations, an LTS approach that does not require a temporal interface equation has been described in which the rapidly incoming information is considered by simply restricting the cell time step to at most four times the minimum time step of the surrounding cells. More recently, Kleb et al.[5] generalized Berger's technique to two-dimensional Euler and Navier-Stokes equations for unstructured grids. To avoid complicated interface equations, Kleb et al.[5] have assigned a distribution of the local time steps that are integers with power-of-two multiples of the global minimum time step, such that for any adjacent time levels the grid is refined by two in time.

Although most of the LTS procedures have been shown to result in significant savings in computer time, little has been written about the major improvement in accuracy that can be obtained via

Received Aug. 11, 1993; revision received Feb. 24, 1994; accepted for publication Feb. 28, 1994.

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